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Pairs of elliptic curves and their Frobenius fields

Abstract: Given an elliptic curve $E$ over a number field $K$. The Frobenius field attached to $E$ at a prime $p$ is the splitting field of the characteristic polynomial of the Frobenius endomorphism acting on the $\ell$-adic Tate module of $E$ ($\ell$ a prime different from $p$) over the rationals. Thus, the splitting field is either of degree 1 or degree 2 over the rationals.

Let $E_1$ and $E_2$ be elliptic curves defined over a number field $K$, with at least one of them without complex multiplication. We prove that the set of places $v$ of $K$ of good reduction such that the corresponding Frobenius fields are equal has positive upper density if and only if $E_1$ and $E_2$ are isogenous over some extension of $K$.

For an elliptic curve $E$ defined over a number field $K$, we show that the set of finite places of $K$ such that the Frobenius field at $v$ equals a fixed imaginary quadratic field $F$ has positive upper density if and only if $E$ has complex multiplication by $F$.

Time permits we will provide a sketch of a result about two dimensional $\ell$-adic Galois representations that we will need using an algebraic density theorem due to Rajan.

EVERYONE IS WELCOME!

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