DEPARTMENT COLLOQUIUM

Mathematics & Computer Sciences

Friday, October 29, 2010

12:00–12:50 in C674

Stephen Curran
Department of Mathematics
University of Pittsburgh at Johnstown
Johnstown, Pennsylvania, USA

Periodic Points of Continuous Functions

A continuous function $f : \mathbb{R} \to \mathbb{R}$ is said to have a *periodic point of period* k if for some $x_0 \in \mathbb{R}$ we have $f^k(x_0) = x_0$, but $f^m(x_0) \neq x_0$ for all positive integers $m < k$.
A theorem of Li and Yorke states that if f has a periodic point of period 3, then f has a periodic point of period k for all positive integers k.
A natural question arises: When does the existence of a periodic point of period k imply the existence of a periodic point of period l?
This question was answered by a theorem of Sharkovsky. One can interpret the search for periodic points of any period from the existence of a periodic point of a given period as the search for a closed walk of a given length in a digraph constructed from the function. This leads to a graph-theoretic proof of Sharkovsky's theorem.