

## NUMBER THEORY & COMBINATORICS SEMINAR

Speaker: Radan Kucera (Masaryk University)  
Date: 2007 Nov 28  
Time: 3:00 – 3:50 pm  
Room: TH 343  
Title: On circular units and the class group of an abelian field

### Abstract:

The aim of this talk is to show that circular units can be used to study the class group of an abelian field. At the beginning, we recall the definition and basic properties of the group of circular units of an abelian number field (explicit generators, finite index in the full group of units, Sinnott's formula containing the class number).

Then we show a concrete application: having a compositum  $K$  of real quadratic fields unramified at 2, we derive a lower bound for the divisibility of the class number of  $K$  by a power of 2. We also explain that circular units can be used to obtain an information concerning the Galois module structure of the class group: for an abelian field  $K$  and a prime  $p$ , two Galois modules appear here naturally, namely the  $p$ -th part of the group of all units modulo the subgroup of circular units and the  $p$ -th part of the class group. Thaine's theorem states that any annihilator of the former module is an annihilator of the latter one, provided  $p$  is odd and relatively prime to the degree of the field.

Finally, a joint result with C. Greither concerning the  $p$ -th part of the class group of cyclic fields of  $p$ -power degree will be explained.