Abstract: Over the past year, Richard Paris (U Abertay Dundee) and myself showed how a modification of a common technique for developing asymptotic expansions of solutions of linear differential equations can be used to derive Hadamard expansions of solutions of differential equations. Hadamard expansions are convergent series of the form

$$
\sum_{n=0}^{\infty} e^{-\lambda_n z} \sum_{k=0}^{\infty} \frac{a_{nk}}{(\rho_n z)^{\mu_n+k}} P(\mu_n + k, \omega_n z), \quad (z \neq 0)
$$

where $P$ is the incomplete gamma function

$$
P(a, z) = \frac{1}{\Gamma(a)} \int_{0}^{z} e^{-t} t^{a-1} dt, \quad (|\arg z| < \pi, \text{Re}(a) > 0).
$$

Such expansions share some of the features of hyperasymptotic expansions, particularly that of having exponentially small remainders when truncated and as a consequence provide a useful computational tool for evaluating special functions.

In this talk, I will describe the usual method of construction of asymptotic solutions to linear differential equations, the modifications needed for obtaining Hadamard expansions, and then in turn show how ridiculously high precision approximate solutions then result through a simple reworking of the Hadamard expansion.

Prepare to enter the world of 20 and more significant figure precision.