



# Lethbridge Number Theory and Combinatorics Seminar

**Wednesday** — January 27, 2016

Room: C630

Time: **10:00 to 10:50 a.m.**

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### The Distribution of Multiplicatively Dependent Vectors

*Abstract:* Let  $n$  be a positive integer,  $G$  be a group and let  $\nu = (\nu_1, \dots, \nu_n)$  be in  $G^n$ . We say that  $\nu$  is a *multiplicatively dependent  $n$ -tuple* if there is a non-zero vector  $(k_1, \dots, k_n)$  in  $\mathbb{Z}^n$  for which  $\nu_1^{k_1} \cdots \nu_n^{k_n} = 1$ .

Given a finite extension  $K$  of  $\mathbb{Q}$ , we denote by  $M_{n,K}(H)$  the number of multiplicatively dependent  $n$ -tuples of algebraic integers of  $K^*$  of naive height at most  $H$  and we denote by  $M_{n,K}^*(H)$  the number of multiplicatively dependent  $n$ -tuples of algebraic numbers of  $K^*$  of height at most  $H$ . In this seminar we discuss several estimates and asymptotic formulas for  $M_{n,K}(H)$  and for  $M_{n,K}^*(H)$  as  $H \rightarrow \infty$ .

For each  $\nu$  in  $(K^*)^n$  we define  $m$ , the *multiplicative rank* of  $\nu$ , in the following way. If  $\nu$  has a coordinate which is a root of unity we put  $m = 1$ . Otherwise let  $m$  be the largest integer with  $2 \leq m \leq n + 1$  for which every set of  $m - 1$  of the coordinates of  $\nu$  is a multiplicatively independent set.

We also consider the sets  $M_{n,K,m}(H)$  and  $M_{n,K,m}^*(H)$  defined as the number of multiplicatively dependent  $n$ -tuples of multiplicative rank  $m$  whose coordinates are algebraic integers from  $K^*$ , respectively algebraic numbers from  $K^*$ , of naive height at most  $H$  and will consider similar questions for them.

**EVERYONE IS WELCOME!**

Visit the seminar web page at <http://www.cs.uleth.ca/~nathanng/ntcoseminar/>