# Lethbridge Number Theory and Combinatorics Seminar

Monday — January 20, 2020
Room: C620
Time: 12:00 to 12:50 p.m.

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# Regular Representations of Groups

A natural way to understand groups visually is by examining objects on which the group has a natural permutation action. In fact, this is often the way we first show groups to undergraduate students: introducing the cyclic and dihedral groups as the groups of symmetries of polygons, logos, or designs. For example, the dihedral group $D_8$ of order $8$ is the group of symmetries of a square. However, there are some challenges with this particular example of visualisation, as many people struggle to understand how reflections and rotations interact as symmetries of a square.

Every group $G$ admits a natural permutation action on the set of elements of $G$ (in fact, two): acting by right- (or left-) multiplication. (The action by right-multiplication is given by $\{\tau_g : g \in G\}$, where $\tau_g(h) = hg$ for every $h \in G$.) This action is called the *right- (or left-) regular representation* of $G$. It is straightforward to observe that this action is regular (that is, for any two elements of the underlying set, there is precisely one group element that maps one to the other). If it is possible to find an object that can be labelled with the elements of $G$ in such a way that the symmetries of the object are precisely the right-regular representation of $G$, then we call this object a *regular representation* of $G$.

A Cayley (di)graph Cay$(G, S)$ on the group $G$ (with connection set $S \subset G$) is defined to have the set $G$ as its vertices, with an arc from $g$ to $sg$ for every $s \in S$. It is straightforward to see that the right-regular representation of $G$ is a subset of the automorphism group of this (di)graph. However, it is often not at all obvious whether or not Cay$(G, S)$ admits additional automorphisms. For example, Cay$(\mathbb{Z}_4, \{1, 3\})$ is a square, and therefore has $D_8$ rather than $\mathbb{Z}_4$ as its full automorphism group, so is not a regular representation of $\mathbb{Z}_4$. Nonetheless, since a regular representation that is a (di)graph must always be a Cayley (di)graph, we study these to determine when regular representations of groups are possible.

I will present results about which groups admit graphs, digraphs, and oriented graphs as regular representations, and how common it is for an arbitrary Cayley digraph to be a regular representation.

**EVERYONE IS WELCOME!**

Visit the seminar web page at
http://www.cs.uleth.ca/∼nathanng/ntcoseminar/