University of Lethbridge
Department of Mathematics and Computer Science

Number Theory & Combinatorics Seminar

Monday — March 11, 2013
Room: E575
Time: 12:00 to 12:50 p.m.

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On sums and products of integers

Abstract: Additive combinatorics has recently attracted a lot of attention in the mathematics world. A famous conjecture in this field, known as Erdős and Szemerédi's conjecture, concerns the sums and products of integers. It asserts the following:

Conjecture. For any fixed $\delta > 0$ the lower bound

$$\max\{|A+A|, |A \cdot A|\} \gg_\delta |A|^{2-\delta}$$

holds for all finite sets $A \subset \mathbb{Z}$.

Here $A + A = \{a + a' : a, a' \in A\}$ and $A \cdot A = \{aa' : a, a' \in A\}$.

Roughly speaking, the conjecture states that: For a fixed a set of integers, both the sum-set and product-set cannot be small. There are two major achievements towards this conjecture which we discuss during the talk:

1- Chang in 2003 showed that the sum-set must be large whenever the product-set is sufficiently small. More precisely, she has shown that

$$|A+A| > 36^{-\alpha}|A|^2$$

if $|A \cdot A| < \alpha|A|$ for some constant α.

2- The best known bound today, achieved by Solymosi, follows from his more general inequality

$$|A+A|^2|A \cdot A| \geq \frac{|A|^4}{2\log|A|}.$$

The Solymosi's result is valid for A a finite subset of the real numbers. From the inequality we have

$$\max\{|A+A|, |A \cdot A|\} \geq \frac{|A|^{4/3}}{2(\log|A|)^{1/3}}.$$

EVERYONE IS WELCOME!

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