GUIDELINES FOR A THESIS IN MATHEMATICS
AT THE UNIVERSITY OF LETHBRIDGE

Writing your thesis is a major part of earning a graduate degree from the University of Lethbridge. We would like to give you some information about this, although nothing we say replaces the university's regulations that are found in the *Master of Science Policies and Procedures* [9, especially §14–§16] and the *Doctor of Philosophy Policies and Procedures* [10, especially §15–§17]. (Valuable advice about writing any mathematics paper, not just a thesis, is provided in [3], and also [2, 4, 5].)

1. BASIC REQUIREMENTS

Your thesis must make a contribution to some field of mathematics, and also report what was previously known about the topic.

- A Ph. D. thesis is expected to have a significant amount of original mathematical research. This will normally include at least one substantial theorem that was not previously known.
- A masters thesis may either be research-based or expository. A research-based thesis has new theorems, like a Ph. D. thesis, but an expository thesis mainly (or entirely) explains results that were already known. To satisfy the university requirement that "the Thesis must consist of original student research work" [9, p. 15], an expository thesis must not only demonstrate good understanding of the chosen topic(s), but also provide new insights, such as a different approach to the subject, additional examples and applications, proofs that were omitted from the source material, or explanations of connections between two different fields.

2. AVOID PLAGIARISM

It is a basic standard of academic writing that every idea that came from someone else must be specifically attributed to the correct source. This is stated in the university's *Graduate Studies Calendar* [8, Part 4, §5b, p. 47]:

> **Plagiarism**
> No student shall represent the words, ideas, images or data of another person as his or her own. This regulation will affect any academic assignment or other component of any course or program of study, whether the plagiarized material constitutes a part or the entirety of the work submitted.

Therefore, you must make sure that your thesis makes a clear distinction between the work of others (such as results, definitions, and applications that were previously known) and the results or ideas that you came up with on your own. Failure to do so is *plagiarism*, which is a very serious academic offence that will result in severe academic penalties, including rejection of the thesis. You will be penalized even if the violation was unintentional (such as due to carelessness).

In particular, you must properly cite the source of every result used in your thesis that is not your own (even if the result has not been published), because: **Any time you state a theorem (or proposition, or lemma, or definition, or other fact or idea) in your thesis without giving a reference, you are telling the reader that it is something new, and you are**

claiming credit for discovering it (or creating it). If it is not actually something new that you created, then you have committed plagiarism.

The only exception to the rule is that common knowledge (information that is widespread and can be expected to be known by the reader) does not require a specific reference. (This includes facts that are typically learned in undergraduate or first-year graduate math courses.) However, if you decide not to include a reference, you should point out that you consider the fact to be common knowledge. For example:

> The following lemma is a well-known consequence of the Inverse Function Theorem.
> *or*
> The following well-known result can be found in many graduate algebra textbooks.
> *or*
> The following folklore theorem has been known to number theorists since at least the 1980's.

Having said that, there is more to avoiding plagiarism than just citing sources. Another basic standard of academic writing is that you can never use the words of another person (not even a sentence, or even a few words if they are distinctive) without putting them in quotation marks (or using a substitute device, such as putting them in an indented block with a different font, as with the above quote from the university calendar). And you must also clearly explain where the words came from. Furthermore, it is not just a person's words: you cannot use a person's ideas without giving them credit. A more detailed explanation of this is provided by the university library [12]:

> **Avoiding Plagiarism**
> It is expected that you consult and use others' research in your writing; however, when you do, you must cite the source of the information. To fail to do so is plagiarism. The following paragraphs should help you determine when you must cite and when a citation isn't necessary.
>
> **When to Cite**
> Whenever you are presenting the words, ideas, images, or data of someone other than yourself, you must cite the source. This includes paraphrases, because even though the words are your own, the idea you're presenting is not. When paraphrasing, ensure that the wording is actually your own; simply rearranging a few of the author's words or replacing them with a few synonyms is not paraphrasing and constitutes plagiarism. When quoting directly, place quotation marks around the author's exact words.
>
> Most citation styles require both an in-text citation (placed immediately following the words, idea, etc. borrowed from another source) and a bibliography or reference list entry at the end of your paper. . . .
>
> **When Not to Cite**
> If the wording is your own and the idea being expressed is your own, no citation is necessary. It is not plagiarism, as you are not presenting the ideas or words of someone else.
>
> In addition, if an idea or fact is widely known and not disputed, it is considered common knowledge and does not need to be cited. This information is generally known by everyone within the discipline and can be found in numerous sources. When unfamiliar with a discipline, as many students are, it can be difficult to know what is common knowledge and what is specialized knowledge that requires a citation. It is always best to err on the side of caution. When in doubt, cite it.

A more thorough discussion of plagiarism can be found in a web module written by M. Roig [7, pp. 1–16]. See [1, Chapter 6, pp. 169–192] for standard guidelines on how to properly incorporate

quoted or paraphrased material, and on the information that needs to be included in each item of your bibliography.

3. How to refer to items in your bibliography

In mathematics, the usual style for referring to the bibliography is to put the item number in square brackets (such as [99]), followed by more specific information, such as the theorem number and page number, which are also inside the brackets. For example:

> A. Zmith [99, Proposition 2.1, p. 5] gave the first example of a nonabelian metric space with uncountably many simple zeroes.
>
> *or*
>
> **Theorem 1** ([99, Proposition 2.1, p. 5])**.** $\mathrm{Hom}(A_1, B_7)$ *is a nonabelian metric space, and has uncountably many simple zeroes.*
>
> *or*
>
> **Theorem 2** (Zmith [99, Proposition 2.1, p. 5])**.** $\mathrm{Hom}(A_1, B_7)$ *is a nonabelian metric space, and has uncountably many simple zeroes.*

The notation [99, Proposition 2.1, p. 5] means that the reader who wants to see the original source should look at Proposition 2.1 on page 5 of item [99] in the bibliography. (The page number can be omitted if the reference is to a short paper, rather than a book, but the reference must be specific enough that the reader can easily find the information that the reference is pointing to.)

Additional examples of citations can be found by looking at math papers, either in journals (in the library or online) or on the arxiv (`http://arxiv.org`), which is the standard repository for (pre-publication) manuscripts in mathematics and some other fields.

4. Quotation marks are rare in mathematical writing

As mentioned above, the standard rule in academia is that if you copy someone's words exactly, then you must put them in quotation marks. However, mathematicians make an exception for technical material, such as the statement of a theorem (or definition). For example, the above theorem of Zmith (that $\mathrm{Hom}(A_1, B_7)$ is a nonabelian metric space, and has uncountably many simple zeroes) does not need quotation marks, even though it may have been copied verbatim from Zmith's paper, as long as the appropriate reference is given. You need to use your judgment to decide whether it is better to copy the theorem word-for-word, or changes are needed so that it is consistent with the notation, terminology, and other content of your paper.

On the other hand, explanations and other commentary *between* the theorems should *not* be copied verbatim. Unlike in some fields (such as the humanities), mathematics papers rarely use direct quotations. Instead, you should think carefully about the *idea* you want to get across, and put it into your own words. If the idea came from someone else (in which case, this is called "paraphrasing"), then, of course, you must give a reference (and include the source in your bibliography). As was already said, you need to make it clear which ideas are your own, and which were learned from someone else.

The writing in a paraphrase needs to be your own, not a modified version of someone else's writing, so it is not sufficient to merely edit the original material by changing a few of the words or symbols — the structure of the sentences and paragraphs should be different. (In order to be able to write an acceptable paraphrase that reproduces the same ideas in different words, you will need to *thoroughly understand* the source material.) This is explained more fully in the above-mentioned web module of Roig [7, pp. 9–14 and 49–63]. The following two quotes provided by Roig [7, p. 8] may also be helpful:

> Simply changing a few words here and there, or changing the order of a few words in a sentence or paragraph, is still plagiarism. (Pechenik, 2001; p. 10).

> You also plagiarize when you use words so close to those in your source, that if your work were placed next to the source, it would be obvious that you could not have written what you did without the source at your elbow. (Booth, Colomb, & Williams, 1995; p. 167)

Your paraphrasing of another author's idea can include the same technical terms (such as *non-abelian finite simple group* or *entire function with no real zeroes*) without quotation marks, but you cannot copy other phrases (such as "by far the best" or "might be necessary for other reasons"), unless you put them in quotation marks.

5. Respect copyright

Copyright law restricts the amount of text that can be copied from a single source, even if it is put into quotation marks and an appropriate citation is given. The law does not provide exact constraints, but, as a guideline to keep in mind, the American Psychological Association [1, 6.10, p. 173] does not allow more than:

> a maximum of three figures or tables from a journal article or book chapter, single text extracts of fewer than 400 words, or a series of text extracts that total fewer than 800 words.

You cannot include more of a copyrighted text than is allowed by law, unless you get permission from the copyright owner (see [9, p. 16] or [10, p. 21]).

You are warned that permission can be refused, and may take a long time even if it is granted. It is much better to write the thesis without copying large sections from other authors. Writing an acceptable thesis is a scholarly activity that involves exploration, interpretation, and creativity, not just pasting together chunks of material copied from other sources.

6. Avoid self-plagiarism

The university's *Graduate Studies Calendar* [8, Part 4, §5d, p. 47] states:

> **Duplication**
> No student shall submit in any course or program of study, without both the knowledge and approval of the person or persons to whom it is submitted, all or a substantial portion of any academic assignment for which credit has previously been obtained or which has been or is being submitted in another course or program of study in the University or elsewhere. (This clause is not intended to prevent the integration of learning but, rather, to prevent duplication of credit for a body of work.)

This means that, for example, you cannot use work you did in an undergraduate research project as part of the original content of your graduate thesis (and your Ph. D. thesis cannot reproduce your Masters thesis).

If your thesis topic is related to work that you did on a thesis (or other research project) elsewhere, so you feel the other work should be included in your discussion of what was previously known, then you need to make it clear that this is being included only as background, not as original content.

On the other hand, it is perfectly acceptable to include results from your published papers if the work was done as part of your current graduate studies. In fact, a thesis is stronger if it includes theorems that have already been published in a reputable journal (or accepted for publication, even if they have not yet appeared in print). Conversely, if your thesis includes new theorems, you are encouraged to also publish them in a journal.

7. LaTeX

You are strongly encouraged to prepare your thesis in LaTeX, which is free typesetting software that is the world-wide standard for mathematics papers. It is installed on the department's Linux

computers (and is available for all major operating systems). Using the department-supplied LaTeX template [11] should make it easier to satisfy the university's detailed formatting requirements that are spelled out in [9, §15] and [10, §16].

The internet has many resources to help you get started with LaTeX. One possibility is [6], and others are listed at

https://www.tug.org/interest.html#latexdoc

(This web page also lists some books that you can buy.) Be sure to learn how to use the `\cite` macro, which is how you make references to your bibliography in LaTeX.

References

[1] American Psychological Association, *Publication Manual of the American Psychological Association,* 6th ed. American Psychological Association, Washington, DC, 2010. (Library call no. BF 76.7 P83 2010)

[2] L. Gillman, *Writing Mathematics Well*, Mathematical Association of America, Washington, D.C., 1987. (Library call no. QA 40 G54 1987)

[3] P. R. Halmos, How to write mathematics. *Enseignement Math.* 16 (1970) 123–152. MR 0277319, `http://dx.doi.org/10.5169/seals-43857`

[4] N. J. Higham, *Handbook of Writing for the Mathematical Sciences,* 2nd ed. Society for Industrial and Applied Mathematics, Philadelphia, PA, 1998. MR 1640787, `http://www.siam.org/books/ot63/`

[5] S. G. Krantz, *A Primer of Mathematical Writing.* American Mathematical Society, Providence, R.I., 1997. `http://bookstore.ams.org/pmw/`

[6] T. Oetiker et al., *The Not So Short Introduction to LaTeX2e.* `http://mirrors.ctan.org/info/lshort/english/lshort-letter.pdf`

[7] M. Roig, Avoiding plagiarism, self-plagiarism, and other questionable writing practices: A guide to ethical writing, *Office of Research Integrity web site* (May 15, 2013). `http://ori.hhs.gov/sites/default/files/plagiarism.pdf`

[8] University of Lethbridge, *Graduate Studies Calendar*, 2015/2016. `http://www.uleth.ca/ross/sites/ross/files/imported/academic-calendar/2016-17/sgs/sgs.pdf`

[9] University of Lethbridge, *Master of Science Policies and Procedures*, Version 1.3 of November 2, 2015. `http://www.uleth.ca/graduate-studies/master-science/policies-procedures`

[10] University of Lethbridge, *Doctor of Philosophy Policies and Procedures*, Version 4.0 of November 2015. `http://www.uleth.ca/graduate-studies/sites/graduate-studies/files/Doctor-of-Philosophy_Policies-and-Procedures.pdf`

[11] University of Lethbridge Department of Mathematics and Computer Science, *Thesis Style.* `http://www.uleth.ca/artsci/math-computer-science/thesis-style`

[12] University of Lethbridge Library, *Understand Plagiarism.* `http://libguides.uleth.ca/plagiarism`

The paper described in the following bibliography item does not exist. It is merely a placeholder to be cited by examples in the above guidelines.

[99] A. Zmith, Title of paper, *Name of Journal*, volume (year) pages.