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On the modularity conjecture  
for abelian varieties over $\mathbb{Q}$

Abstract: The modularity theorem tells us that for every elliptic curve $E$ over $\mathbb{Q}$, there is a modular form $f_E$ such that the L-function $L(s, f_E)$ for $f_E$ coincides with the L-function $L(s, \rho_E)$ for the Galois representation on the Tate module of $E$. In fact, $f_E$ is a cusp form and its level is determined by the conductor of $\rho_E$. Since the modular form $f_E$ determines an automorphic representation $\pi_E$ of $GL_2(\mathbb{A}_\mathbb{Q})$ with the same L-function as $f_E$, we have

$$L(s, \rho_E) = L(s, \pi_E).$$

The modularity conjecture for abelian varieties is the obvious generalization of this theorem from the case of one-dimensional abelian varieties: For every abelian variety $A$ over $\mathbb{Q}$ there is an automorphic representation $\pi_A$ of a group $G(\mathbb{A}_\mathbb{Q})$ such that

$$L(s, \rho_A) = L(s, \pi_A),$$

where $\rho_A$ is the Galois representation on the Tate module of $A$.

In this talk I will describe recent joint work with Lassina Dembélé giving new instances of the modularity conjecture for abelian varieties over $\mathbb{Q}$. Where do we hunt for $\pi_A$? What is the group $G$ over $\mathbb{Q}$? What is the level of $\pi_A$? Can we find a generalized modular form $f_A$ from which $\pi_A$ can be built? I will explain how we use work of Benedict Gross, Freydoon Shahidi and others to answer these questions. I will also explain how thesis work by Majid Shahabi illuminates the level of $\pi_A$.

This is joint work with Lassina Dembélé.

EVERYONE IS WELCOME!

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