Lethbridge Number Theory and Combinatorics Seminar

Monday — December 4, 2017
Room: C630
Time: 12:00 to 12:50 p.m.

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On the modularity conjecture for abelian varieties over $\mathbb{Q}$

Abstract: The modularity theorem tells us that for every elliptic curve E over $\mathbb{Q}$, there is a modular form f_E such that the L-function $L(s, f_E)$ for f_E coincides with the L-function $L(s, \rho_E)$ for the Galois representation on the Tate module of E. In fact, f_E is a cusp form and its level is determined by the conductor of ρ_E. Since the modular form f_E determines an automorphic representation π_E of $\mathrm{GL}_2(\mathbb{A}_\mathbb{Q})$ with the same L-function as f_E, we have

$$L(s, \rho_E) = L(s, \pi_E).$$

The modularity conjecture for abelian varieties is the obvious generalization of this theorem from the case of one-dimensional abelian varieties: For every abelian variety A over $\mathbb{Q}$ there is an automorphic representation π_A of a group $G(\mathbb{A}_\mathbb{Q})$ such that

$$L(s, \rho_A) = L(s, \pi_A),$$

where ρ_A is the Galois representation on the Tate module of A.

In this talk I will describe recent joint work with Lassina Dembélé giving new instances of the modularity conjecture for abelian varieties over $\mathbb{Q}$. Where do we hunt for π_A? What is the group G over $\mathbb{Q}$? What is the level of π_A? Can we find a generalized modular form f_A from which π_A can be built? I will explain how we use work of Benedict Gross, Freydoon Shahidi and others to answer these questions. I will also explain how thesis work by Majid Shahabi illuminates the level of π_A.

This is joint work with Lassina Dembélé.

EVERYONE IS WELCOME!

Visit the seminar web page at
http://www.cs.uleth.ca/∼nathanng/ntcoseminar/

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