Lethbridge Number Theory and Combinatorics Seminar

Monday — November 20, 2017
Room: C630
Time: 12:00 to 12:50 p.m.

Forrest Francis

Euler's Function on Products of Primes in Progressions

Abstract: Let $\phi(n)$ be Euler's totient function and let q and a be fixed coprime natural numbers. Denote by $S_{q,a}$ the set of natural numbers whose prime divisors are all congruent to a modulo q. We can establish

$$\limsup_{n \in S_{q,a}} \frac{n}{\phi(n)\big(\log(\phi(q)\log n)\big)^{1/\phi(q)}} = \frac{1}{C(q,a)},$$

where $C(q,a)$ is a constant associated with a theorem of Mertens. We may then wish to know whether there are infinitely many n in $S_{q,a}$ for which

$$\frac{n}{\phi(n)\big(\log \phi(q)\log n\big)^{1/\phi(q)}} > \frac{1}{C(q,a)} \tag{1}$$

is true. In the case $q = a = 1$, Nicolas (1983) established that if the Riemann hypothesis is true, then (1) holds for all primorials (products of the form $\prod_{p \le x} p$), but if the Riemann hypothesis is false then there are infinitely many primorials for which (1) is true and infinitely many primorials for which (1) is false.

In this talk we will show that, for some $q > 1$, the work of Nicolas can be generalized by replacing the Riemann hypothesis with analogous conjectures for Dirichlet L-functions and replacing the primorials with products of the form

$$\prod_{\substack{p \le x \\ p \equiv a \,(\mathrm{mod}\, q)}} p.$$

EVERYONE IS WELCOME!

Visit the seminar web page at
http://www.cs.uleth.ca/~nathanng/ntcoseminar/

Pacific Institute *for the* Mathematical Sciences