Apodization Functions for Fourier Transform Spectroscopy

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Abstract: The three Norton-Beer apodizing functions provide a reduction in the sidelobe amplitude of FTS instrumental line shapes. We extend this series of apodizing functions to provide line shapes with FWHM ranging from 1.1 to 2.0.

OCIS codes: (300.6300) Spectroscopy, Fourier transforms; (300.3700) Linewidth

1. Introduction

It is common practice in Fourier transform spectroscopy to multiply the measured interferogram by an apodizing function in order to reduce the amount of ringing present in the resulting instrumental line shape (ILS) [1]. Many apodizing functions have been reported in the literature [2,3,4,5] and practitioners often make their choice without a clear understanding of the role of the function on the independence of the resulting spectral data points [6]. While purists would question the need for apodization in the first place, the reduction in the amplitude of the secondary maxima of the ILS, albeit at the cost of lower spectral resolution, is often desired. In this paper we extend upon the work of Norton and Beer [3] to generate a family of apodizing functions that are close to optimum, in the sense that, to a large degree, they preserve the orthogonal properties of the sinc function, provide near optimum reduction in the amplitude of secondary maxima for a given decrease in spectral resolution, and are simple to compute.

2. Background

During his extensive study, Filler [2] devised a graphical method for comparing different apodizing functions and their corresponding ILSs. This method graphs the normalized height of the absolute largest secondary lobe of the ILS, relative to the height of the absolute largest secondary lobe of the sinc function, against the full width at half maximum (FWHM) of the ILS, again relative to the FWHM of the sinc function.

Filler [2] introduced two families of apodizing functions, $D_\alpha(\delta)$ and $E_\alpha(\delta)$ (where δ is the optical path difference out to a maximum value of L) defined as:

$$\begin{aligned} D_\alpha(\delta) &= \cos\left(\frac{\pi\delta}{2L}\right) + \alpha\cos\left(\frac{3\pi\delta}{2L}\right) & 0 \le \alpha \le 1 \\ E_\alpha(\delta) &= 1 + (1+\alpha)\cos\left(\frac{\pi\delta}{L}\right) + \alpha\cos\left(\frac{2\pi\delta}{L}\right) & 0 \le \alpha \le 1 \end{aligned} \tag{1}$$

which he considered to give superior performance to other commonly used functions. Norton and Beer [3,4] extended this analysis and introduced the functions, $P_{\alpha,p}(\delta)$, variants of the $E_\alpha(\delta)$ family, where:

$$P_{\alpha,\,p}(\delta) = 1 + p + (1+\alpha)\cos\left(\frac{\pi\delta}{L}\right) + \alpha\cos\left(\frac{2\pi\delta}{L}\right) \qquad -1 \le \alpha \le 1\,;\, 0 \le p \le 1 \tag{2}$$

While the $P_{\alpha,p}(\delta)$ functions were judged to be superior to $D_\alpha(\delta)$ and $E_\alpha(\delta)$, by their locus on the Filler diagram, their convergence was rather slow. This provided the impetus for Norton and Beer to explore other families of apodizing functions, which led them to the generic form:

$$NB\left(\frac{\delta}{L}\right)=\sum_{i=0}^{n}C_i\left(1-\left(\frac{\delta}{L}\right)\right)^i \qquad -1\leq\alpha\leq 1\,;\,0\leq p\leq 1 \tag{3}$$

As a result of this work, Norton and Beer introduced three functions corresponding to weak, medium and strong apodization that produced near optimal reduction in the amplitude of the sidelobes for increases in the FWHM of the ILS corresponding to 20, 40 and 60%, respectively. They found that the locus of these functions when plotted on the Filler diagram could be empirically described by an imaginary line and issued a challenge to the mathematically minded to prove that such a boundary exists.

3. Extended apodizing functions

In this paper, we extend the work of Norton and Beer to generate 10 apodizing functions which correspond to FWHM of the ILS ranging from 1.1 to 2.0 in steps of 0.1. Seven of these functions are new; three represent minor changes to those given earlier. Fig. 1 shows the resulting ILS from the 10 functions compared with the sinc ILS.

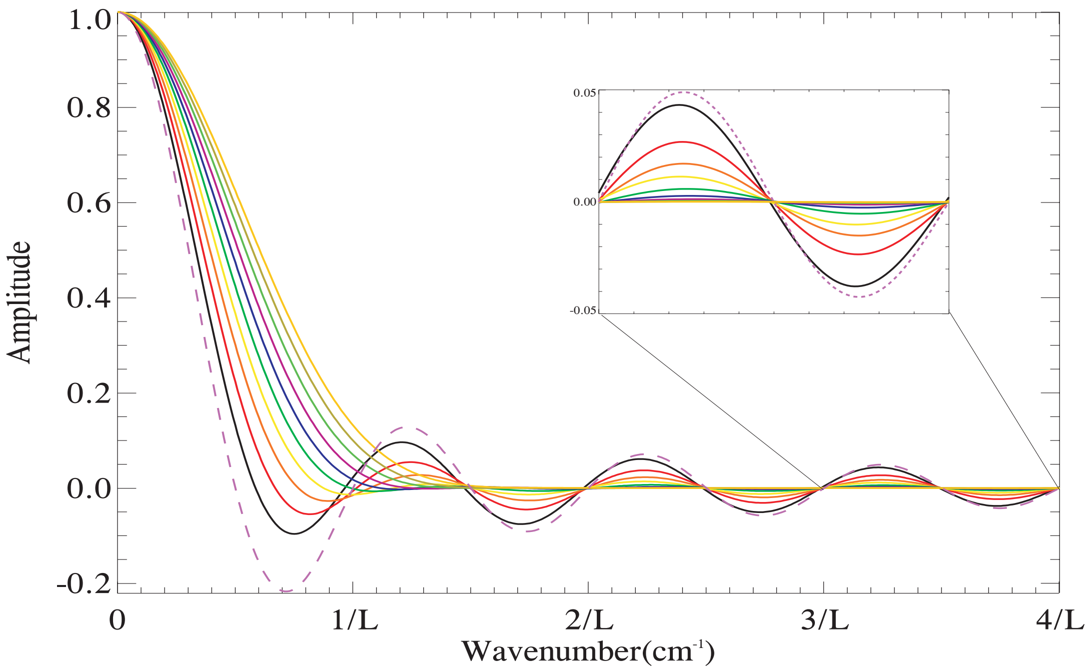

Fig. 1. Instrumental line shapes corresponding to the 10 apodizing functions described in the text compared with the sinc function (dashed line). Inset shows the magnified region of the third sidelobe.

Fig. 2 shows the locus of the 10 apodizing functions on the Filler diagram (red circles) compared with the three original ones of Norton and Beer (blue circles) corresponding to FWHM of 1.2, 1.4 and 1.6. Also shown is the location of the triangle (Bartlett) apodizing function, which although frequently used is seen to be far from optimum. The solid line is the empirical fit that Norton and Beer found to represent the optimum boundary.

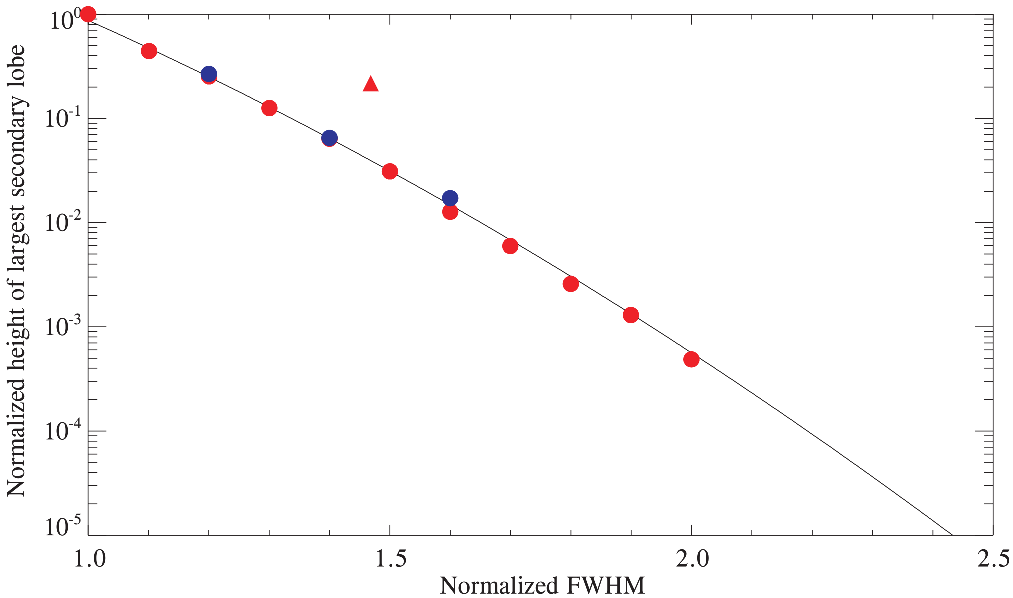

Fig. 2. Filler diagram of the 10 apodizing functions described in this paper (red circles) and the three original ones of Norton and Beer (blue circles). Also shown is the locus of the commonly used triangle apodizing function (triangle).

4. Conclusion

We have extended the work of Norton and Beer to introduce 10 apodizing functions which correspond to FWHM of the ILS ranging from 1.1 to 2.0 in steps of 0.1. The methods used to determine the coefficients of these functions (Eqn. 1) will be presented in a forthcoming paper [7]. When displayed on the Filler diagram, the new functions are found to support the concept of the empirical boundary previously determined. The functions can be used to study the trade-off between ringing in the ILS and spectral resolution, and have application in diverse fields involving Fourier analysis.

5. References

1. R.J. Bell, 'Introductory Fourier Transform Spectroscopy', (Academic Press, New York, 1972).
2. A. S. Filler, "Apodization and Interpolation in Fourier-Transform Spectroscopy", J. Opt. Soc. Am. **54**, 762-767 (1964).
3. R. H. Norton and R. Beer, "New Apodizing Functions for Fourier Spectrometry", J. Opt. Soc. Am. **66**, 259-264 (1976).
4. R. H. Norton and R. Beer, "Errata - New Apodizing Functions for Fourier Spectrometry", J. Opt. Soc. Am. **67**, 419 (1977).
5. F. J. Harris, "On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform," Proceedings of the IEEE **66**, 51-83 (1978).
6. P. Fellgett in printed comments on R. Beer and A.H. Cayford, J. Phys. (Paris) **28**, C2 - 33 (1967).
7. D.A. Naylor and M.T. Tahic, in preparation.