Increased Efficiency through Undersampling in Fourier Transform Spectroscopy

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Abstract: We present the use of undersampling with a narrow band FTS operating in the step-and-integrate mode. Spectra can be unambiguously retrieved from interferograms sampled at less than the DC band-limited Nyquist sampling interval.

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1. Introduction

A primary limitation of Fourier Transform Spectrometers (FTS) is the time required to acquire an interferogram. This is particularly problematic in imaging spectroscopic applications, as well as in applications such as submillimetre astronomy where the dominant noise component arises from changes in atmospheric emission and transmission during the acquisition of an interferogram (typically occurring at frequencies ≥ 1 Hz). In order to minimize the effects of atmospheric variations, or to perform hyperspectral imagery of moving targets, it is imperative to acquire data as quickly as possible.

1.1. Rapid-Scan mode

Astronomical FTS systems are commonly operated in a rapid-scan (RS) mode, in which an interferogram of a source is obtained as quickly as possible, followed by one of a nearby background position. However, since a pair of full resolution scans takes about one minute, atmospheric stability is likely to be a limiting factor. When operating with both input ports viewing the sky, an FTS measures the difference between radiation entering each port and thus effectively cancels out the effects of variations in atmospheric emission. Signal fluctuations due to variations in atmospheric *transmission* during a scan remain, however, and cannot be corrected.

1.2. Step-and-Integrate mode

In an alternate operating mode, known as step-and-integrate (SI), the optical path difference (OPD) in the interferometer is incremented in discrete steps, and the signal is integrated only when the interferometer mirrors are stationary. The signal at each step is modulated by external means so that the detector alternately views source and background. The modulated signal is detected with a lock-in amplifier, and the noise bandwidth in the SI mode is proportional to the reciprocal of the time constant of the lock-in (typically ~ 1 Hz). This is significantly less than the noise bandwidth in the RS mode (typically ~ 1 kHz), which in principle can lead to an increase in overall sensitivity. The main problem with the SI mode is that it takes much longer to acquire an interferogram. At submillimetre wavelengths, through the use of narrowband optical filters which are matched to regions of low atmospheric opacity, we have shown that it is possible to sample the interferogram at less than the interval determined from the DC band limited Nyquist frequency (a condition known as undersampling) and still unambiguously recover the spectral information [1].

The main drawback of the SI mode is that it takes ~ 30 minutes to acquire an interferogram; this is due to the finite speed of the chopping optics, the lock-in time constant and the point-to-point motion time of the interferometer stage. While the SI mode effectively removes atmospheric emission variations on short timescales, the variation of atmospheric *transmission* during the relatively long time it takes to complete a high resolution scan remains a problem. The SI mode outperforms the RS mode when extracting the continuum component of emission because this information is encoded around the zero optical path difference region of the interferogram; a region that is measured, differentially, on much shorter timescales in the SI mode.

2. Undersampled step-and-integrate technique

From information theory the highest frequency component that can be unambiguously recovered is determined by the Nyquist sampling theorem [2]:

$$\Delta z \leq \frac{1}{2 \cdot \sigma_n} \quad (1)$$

where Δz (cm) is the OPD sampling interval and σ_n is known as the Nyquist frequency (cm^{-1}). Implicit in this relation is the assumption that there are signal components spanning the interval from 0 to σ_n cm^{-1}. In the case of a band-limited signal, the minimum OPD sampling interval is given by the Shannon sampling theorem [3]:

$$\Delta z \leq \frac{1}{2 \cdot (\sigma_{max} - \sigma_{min})} \quad (2)$$

where σ_{min} and σ_{max} are the frequency limits of the signal band.

The upper left plot in Fig. 1 shows the simulated 450 μm interferogram for a dual-band (450 / 850 μm) astronomical FTS [4] under construction for the James Clerk Maxwell Telescope (JCMT); the upper right plot shows the corresponding atmospheric transmission spectra multiplied by the filter characteristics [5]. Also shown in this figure are the interferogram sample points needed to recover these spectra based on the Nyquist criteria. The sampling is driven by the highest frequency component (for the 450 μm band, Δz = 0.02 cm OPD, σ_n = 25 cm^{-1}). Although the 850 μm filter is narrower and has a lower upper bandpass limit than the 450 μm filter, it is the 450 μm filter that sets the sampling interval since both signals are sampled simultaneously. Based on the width of the 450 μm filter and the Shannon sampling theorem, the signal bandwidth is ~3 cm^{-1}, which corresponds to a maximum sampling interval of ~0.16 cm OPD. Increasing the sampling interval from that determined by the DC band limited Nyquist frequency case, however, results in aliasing of the spectrum. Since the FTS observes both the 850 and 450 μm bands simultaneously, the challenge is to find an optical path sampling interval that will alias both bands such that neither loses spectral integrity. The lower left plot shows one possible solution; by choosing a sampling interval of Δz = 0.1 cm OPD, corresponding to a Nyquist frequency of 5 cm^{-1}, it can be seen that spectra from both bands can be retrieved with integrity. This technique provides at least a factor of five reduction in the required sampling rate over a DC band-limited measurement, which translates to a factor of five decrease in scan time for the undersampled step-and-integrate mode.

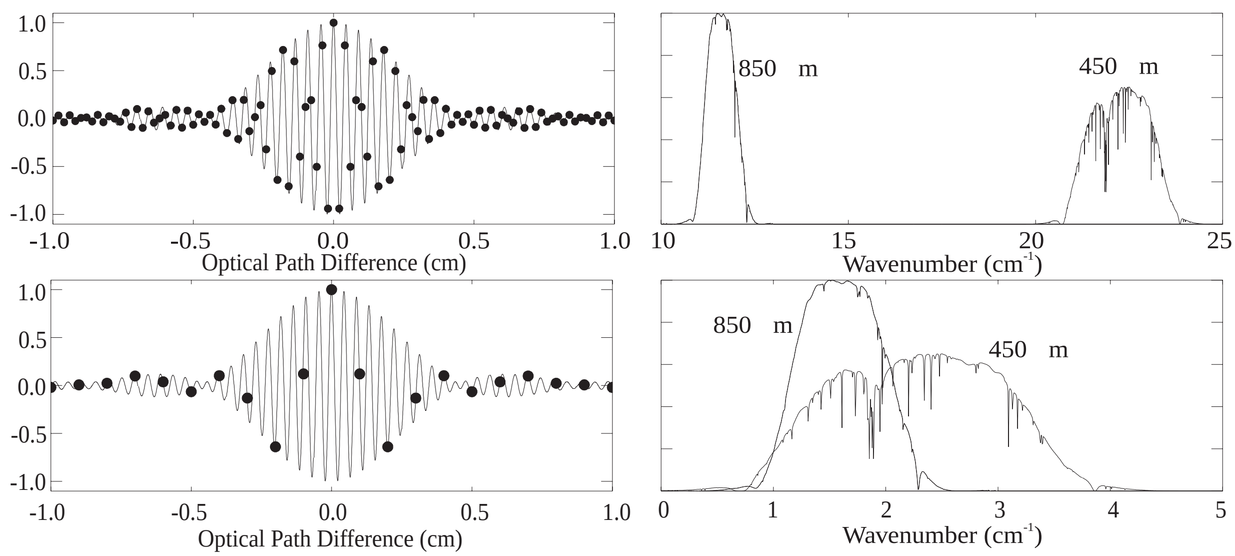

Fig. 1. Top Left: Interferogram for the 450 μm band; symbols show the DC band-limited Nyquist samples (σ_n = 25 cm^{-1}). Top Right: The corresponding 450 and 850 μm atmospheric transmission spectra. Bottom Left: Interferogram for the 450 μm band; symbols represent the samples corresponding to σ_n of 5 cm^{-1} (undersampling). Bottom Right: The recovered 450 and 850 μm spectra retain their spectral integrity

3. Results

The rapid-scan and undersampled step-and-integrate modes of operation were compared in two observing runs at the JCMT in October 2003 and March 2004. Our principal target was the core of the Orion Molecular cloud (OMC1), a bright astronomical source, which has been observed with the JCMT to have a flux of ~167 Jy in the 850 μm band (only a small fraction of the flux from the atmosphere). High spectral resolution scans of OMC1 (Δσ = 0.005 cm^{-1}) were obtained in the undersampled step-and-integrate and rapid-scan modes under the same atmospheric conditions. The total integration time was the same for both the SI and RS observations. The time taken for a full resolution SI spectrum was ~30 minutes; this time could be reduced significantly by having an FTS specifically designed for rapid point-to-point moves and by increasing the modulation frequency.

The spectrum obtained from the Fourier transform of the phase corrected, undersampled, step-and-integrate interferogram can be compared with the equivalent spectrum determined from the average of the corresponding rapid-scan spectra. The two resulting spectra are compared in Fig. 2. Although the observing conditions rendered the quality of these data very poor, it is clear that many spectral lines can still be readily identified in both spectra. The lower trace in the top panel of this figure corresponds to the undersampled SI spectrum; the upper trace to the RS spectrum (offset for clarity). Both spectra show a clear continuum component, with the undersampled SI spectrum appearing to have better signal-to-noise. In order to quantitatively compare the noise between the two methods, it is instructive to look at the signal outside of the bandpass of the optical filter. The lower panel in Fig. 2 shows the noise in the 13 to 14 cm^{-1} region for the undersampled SI spectrum (upper trace) and corresponding rapid-scan spectrum (lower trace), again offset for clarity. The reduction in noise in the undersampled SI mode is evident.

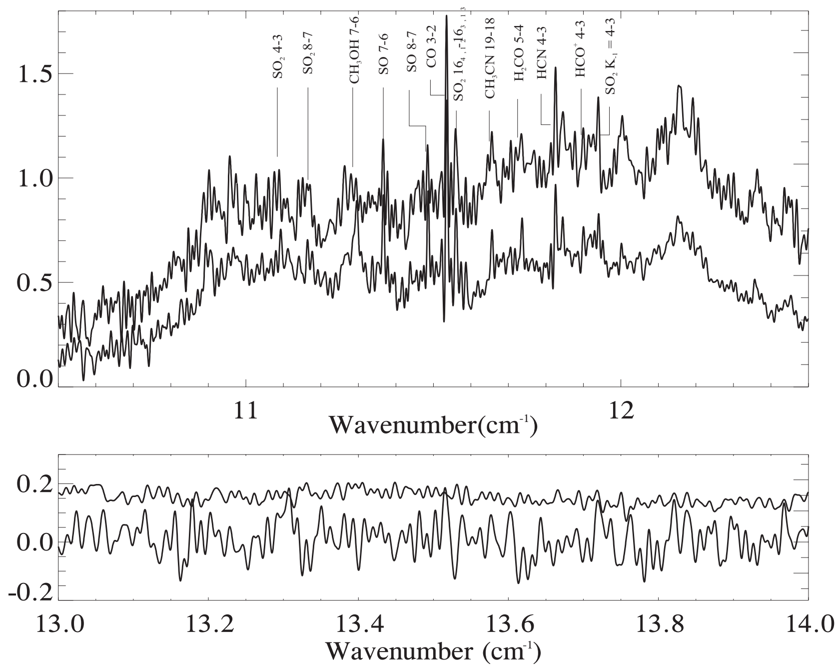

Fig. 2. The top panel shows a comparison between the step-and-integrate spectrum of Orion (lower trace) and the corresponding rapid-scan spectrum (upper trace), offset for clarity. The bottom panel shows the out-of-band noise for the step-and-integrate spectrum (upper trace) and corresponding rapid-scan spectrum (lower trace), again offset for clarity.

4. Conclusions

In this paper we have presented initial results from a Fourier transform spectrometer operated in the undersampled, SI mode. This mode is only made possible through the use of narrowband optical filters, with high out-of-band rejection. Comparison with spectra obtained in an equivalent time with the RS mode clearly shows the advantages and potential of this technique. The undersampled SI mode has been shown [1] to outperform the rapid scan mode when it comes to extracting the continuum component of emission because this information is encoded around the zero optical path difference region of the interferogram, which is measured, differentially, on much shorter timescales. We plan to repeat these measurements under, hopefully, better observing conditions in October 2004, as well as apply the technique to imaging FTS systems currently under development.

5. References

1. D.A. Naylor, B.G. Gom, M.K. Tahic, and G.R. Davis, "Astronomical spectroscopy using an aliased, step-and-integrate, Fourier transform spectrometer," in Astronomical Telescopes and Instrumentation, J. Zmuidzinas and W. S. Holland, eds., Proc SPIE 5498, in press (2004).
2. H. Nyquist, "Certain topics in telegraph transmission theory," Trans. AIEE **47**, 617-644 (1928).
3. C.E. Shannon, "Communication in the presence of noise," Proc. Institute of Radio Engineers **37**, 10-21 (1949).
4. B.G. Gom and D.A. Naylor, "An update on the imaging Fourier transform spectrometer for SCUBA-2," in Astronomical Telescopes and Instrumentation, J. Zmuidzinas and W. S. Holland, eds., Proc SPIE 5498, in press (2004).
5. W.S. Holland et. al., "SCUBA: a common-user submillimetre camera operating on the James Clerk Maxwell Telescope," MNRAS **303**, 659-672 (1999).