Lethbridge Number Theory and Combinatorics Seminar

Monday — January 21, 2019
Room: D631
Time: 12:00 to 12:50 p.m.

Amir Akbary

University of Lethbridge

Ambiguous Solutions of a Pell Equation

It is known that if the negative Pell equation $X^2 - DY^2 = -1$ is solvable (in integers), and if (x, y) is its solution with smallest positive x and y, then all of its solutions (x_n, y_n) are given by the formula

$$x_n + y_n\sqrt{D} = \pm(x + y\sqrt{D})^{2n+1}$$

for $n \in \mathbb{Z}$. Furthermore, a theorem of Walker from 1967 states that if the equation $aX^2 - bY^2 = \pm 1$ is solvable, and if (x, y) is its solution with smallest positive x and y, then all of its solutions (x_n, y_n) are given by

$$x_n\sqrt{a} + y_n\sqrt{b} = \pm(x\sqrt{a} + y\sqrt{b})^{2n+1}$$

for $n \in \mathbb{Z}$. We describe a unifying theorem that includes both of these results as special cases. The key observation is a structural theorem for the non-trivial ambiguous classes of the solutions of Pell equations $X^2 - DY^2 = \pm N$. This talk is based on the work of Forrest Francis in an NSERC USRA project in summer 2015.

EVERYONE IS WELCOME!

Visit the seminar web page at
http://www.cs.uleth.ca/∼nathanng/ntcoseminar/

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